1.) ALGEBRA

→ Laws of Indices

$\boxed{a^m \times a^n = a^{m+n}}$ $\boxed{(a^m)^n = a^{mn}}$ $\boxed{\frac{a^m}{a^n} = a^{m-n}}$

$\boxed{a^{\frac{1}{m}} = \sqrt[m]{a}}$ $\boxed{a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m}$ $\boxed{a^{-m} = \frac{1}{a^m}}$

$\boxed{a^0 = 1}$ $\boxed{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}}$ $\boxed{(ab)^n = a^n . b^n}$

Eg. Solve for x

$2^{x+1} = 8$

* make sure the base is the same

$\therefore 8 = 2^3 \Rightarrow 2^{x+1} = 2^3 \quad \therefore x+1 = 3 \Rightarrow \underline{\underline{x = 2}}$

⇒ Surds

$\boxed{\sqrt{ab} = \sqrt{a}\sqrt{b}}$ $\boxed{\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}}$ $\boxed{a = (\sqrt{a})^2 = \sqrt{a}\sqrt{a}}$

◉ Rationalising

* Remember: $\boxed{(x+a)(x-a) = x^2 - a^2}$

Eg $\frac{1}{1+\sqrt{2}}$

* multiply the top & bottom by the DENOMINATOR, but change the sign

$$\frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} \rightarrow \frac{1-\sqrt{2}}{(1+\sqrt{2})(1-\sqrt{2})} \rightarrow \frac{1-\sqrt{2}}{1-2} = \frac{1-\sqrt{2}}{-1} = \underline{\underline{-1+\sqrt{2}}}$$

Eg2 $\frac{1}{\sqrt{3}}$

$$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \underline{\underline{\frac{\sqrt{3}}{3}}}$$

⇒ Algebraic Fractions & Simplifying Expressions

Eg. $4x + \frac{4x}{x+1} - 4(x+1)$

* Find the common denominator, by taking All denominators and multiplying. In this case it would be $1 \times (x+1) \times 1 = x+1$

$$= \frac{4x(x+1)}{x+1} + \frac{4x}{x+1} - \frac{4(x+1)(x+1)}{x+1}$$

$4(x+1)^2 = 4(x^2+2x+1)$
$= 4x^2+8x+4$

* Always remember negative signs

$$= \frac{4x^2+4x+4x-4x^2-8x-4}{x+1}$$

$$= -\frac{4}{x+1}$$

* To Simplify, Combine into one fraction first, if possible

2.) Quadratics & Polynomials

⇒ Sketching Quadratic graphs

* They are always U-shaped or ∩-shaped. → $\boxed{(+ve) \rightarrow (-ve)}$
* find where they cross the axis → $\boxed{y=0}$ & $\boxed{x=0}$ → (x,y)
* Find minimum or maximum points → this is the turning point

Eg. $y = 2x^2 - 4x + 3$

* This is U shaped because (x^2) is +ve – positive

* $x = 0$

$y = 2(0) - 4(0) + 3$

$\therefore y = 3 \rightarrow (0,3)$

* $y = 0$

$2x^2 - 4x + 3 = 0$

* Use $b^2 - 4ac$ to see if it has any roots (ie. if it actually crosses the x axis)

$b^2 - 4ac = (-4)^2 - 4(2)(3) = -8$

$\therefore$ since $b^2 - 4ac < 0$ there are no roots.

* Vertex (or turning Point)

$y = 2x^2 - 4x + 3$ → Complete the Square

$y = 2(x-1)^2 + 1$

* take the ~~coefficient~~ factor of x^2 out

$2(x^2 - 2x) + 3$

* change to one braket squared

$2(x-1)^2 + d$ → d is what is needed to equate the equations

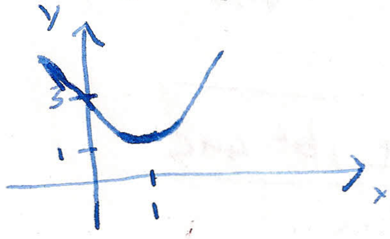
* $\therefore 2(x-1)^2 + d = 2x^2 - 4x + 3$

$\cancel{2x^2} - \cancel{4x} + 2 + d = \cancel{2x^2} - \cancel{4x} + 3$

$\therefore d = 1$

$\therefore$ min Value is where $y = 1$, this is where $x = 1$

$y = 2(1-1)^2 + 1$



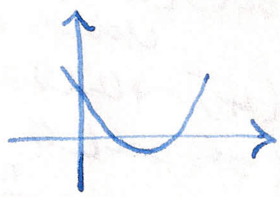
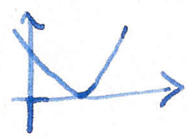
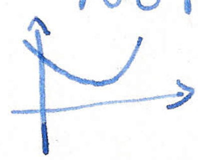
⇒ Completing the Square formula

$$ax^2+bx+c = a\left[x+\frac{b}{2a}\right]^2 + d$$

⇒ Quadratic formula

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

⇒ Roots.

$\boxed{b^2-4ac>0}$	$\boxed{b^2-4ac=0}$	$\boxed{b^2-4ac<0}$
Two roots	one root	No roots
		

⇒ Proved!

$ax^2+bx+c=0$ → Complete the square
→ Aim is to get x on one side

$\rightarrow a\left(x^2+\frac{b}{a}x\right)+c=0$

$\Rightarrow a\left(x+\frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0$

$\rightarrow \left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

$\rightarrow x+\frac{b}{2a} = \sqrt{\frac{b^2-4ac}{4a^2}}$ ⟶ $\sqrt{4a^2} = 2a$

$\therefore x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

3.) Simultaneous equations & Inequalities

E.g. $x-3 < -1+2x$

$x-2 < 2x$

$-2 < x$ or $x > -2$

* $<$ - less than
  $>$ - greater than
  $\leq$ - less than or equal to
  $\geq$ - greater than or equal to

* Direction of inequality changes by dividing/multiplying both sides with a Negative Number

eg. $4-3x \leq 16$

$-3x \leq 12$

$= \underline{x \geq -4}$ [OR $-4 \leq x$]

⇒ Quadratic Inequality

$-x^2+2x+4 \geq 1$ * make '0' on one side

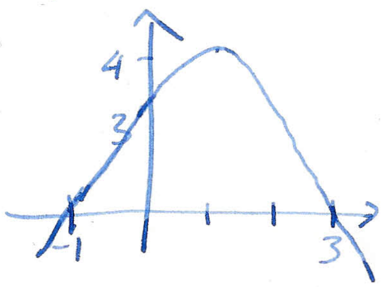
$-x^2+2x+3 \geq 0$ * Draw $y = -x^2+2x+3$

$= x^2-2x-3$

$= (x+1)(x-3) = 0$

$\Rightarrow x = -1$ or $x = 3$

* You need $\geq 0 \Rightarrow$ Positive or zero
  i.e. when it's above the x-axis.



* This is when x is between -1 & 3

$\therefore -x^2+2x+4 \geq 1$ when $x \leq 3$, $x \geq -1$ or $\underline{-1 \leq x \leq 3}$

⇒ Simultaneous Equations With Quadratics

Eg. $-x+2y=5$

$x^2+y^2=25$

* make x or y on their own

Eg. $2y-5=x$ or $y=\frac{x+5}{2}$

* Substitute into quadratic

$(2y-5)^2+y^2=25$

$4y^2-20y+\cancel{25}+y^2=\cancel{25}$

$5y^2-20y=0$

$5y(y-4)=0 \quad \therefore \underline{\underline{y=0 \text{ or } y=4}}$

* Solve for the other variable

$2(0)-5=x$ or $2(4)-5=x$

$\underline{x=-5}$ $\underline{x=3}$

4.) Coordinate Geometry & Graphs

⇒ Equation of a line through two points

$\boxed{y - y_1 = m(x - x_1)}$ $\boxed{y = mx + c}$ $\boxed{ax + by + c = 0}$

* $\boxed{m = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}}$

E.g. $\underset{(x_1, y_1)}{(-3, 10)}$; $\underset{(x_2, y_2)}{(1, 4)}$ * Label points

Gradient $= m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 10}{1 - (-3)} = \dfrac{-6}{4} = \dfrac{-3}{2}$ * Find gradient

* $\boxed{y - y_1 = m(x - x_1)}$ * Write equation always.

$\Rightarrow y - 10 = \dfrac{-3}{2}(x - (-3))$

$\Rightarrow 2y - 20 = -3x - 9$ $\boxed{* \; ax + by + c = 0}$ OR as $y = mx + c$

$2y + 3x - 11 = 0$ or $3x + 2y - 11 = 0$

⇒ Parallel lines have Equal Gradients

eg. $3x - 4y - 7 = 0$ Find the parallel passing through $A(3, -1)$

* $y = mx + c$

$y = \frac{3}{4}x - \frac{7}{4}$ $\therefore$ new equation is $y = \overset{\text{same gradient}}{\frac{3}{4}}x + c$

*Insert $A(3, -1)$ $\Rightarrow -1 = \frac{3}{4}(3) + c$

$-4 = 9 + 4c \therefore c = -\frac{13}{4}$

$\Rightarrow \therefore y = \frac{3}{4}x - \frac{13}{4}$

→ Perpendicular line is gradient $-1 \div$ other gradient
→ also known as 'normals'

$$\boxed{\underset{m_1}{\text{Original gradient}} \times \underset{m_2}{\text{Perpendicular gradient}} = -1}$$

Eg. $3x - 4y - 7 = 0$ $\qquad$ $B(-2,4)$

$y = \frac{3}{4}x - \frac{7}{4}$ $\qquad$ $\therefore$ Perpendicular gradient $= \frac{-1}{\frac{3}{4}} = \frac{-4}{3}$

$y = \frac{-4}{3}x + c$ $\qquad$ # Insert B

$4 = \frac{-4}{3}(-2) + c$

$12 = 8 + 3c$

$\therefore c = \frac{4}{3}$ $\qquad$ $\therefore \Rightarrow \underline{\underline{y = \frac{-4}{3}x + \frac{4}{3}}}$

5.) Sequences & Series

$$n^{th}\text{ term} = \underbrace{a}_{1^{st}\text{ term}} + (\underbrace{n}_{\text{position of any term}} - 1)\underbrace{d}_{\text{common difference}}$$

Eg. 2, 5, 8, 11.... 20^{th} term

$a = 2$; $d = (5-2) = 3$

$\therefore 20^{th}$ term $= a + (n-1)d$

$= 2 + (20-1)3$

$= \underline{59}$

General term $= 2 + (n-1)3$

$= 2 + 3n - 3$

$= \underline{3n-1}$

$$\underbrace{S_n}_{\text{total of the first } n \text{ terms}} = n \times \left(\frac{a + l}{2}\right) \quad l \to \text{last value}: \; l = a + (n-1)d$$

Eg. Sum of 1^{st} term $= 3$, last $= 87$; $d = 4$

* $a = 3$; $l = 87$, $d = 4$ but what is n?

$l = a + (n-1)d$ *make sure you have a, l, d & n.

$87 = 3 + (n-1)4$

$\therefore n = 22$

$\therefore S_{22} = 22 \times \left(\frac{3+87}{2}\right) = \underline{990}$

$\rightarrow \ l = a + (n-1)d$

$$S_n = n \times \left(\frac{a+l}{2}\right)$$

$$\therefore S_n = n \times \frac{a + (a + (n-1)d)}{2}$$

$$S_n = \frac{2an + (n-1)dn}{2}$$

$* \ \frac{n}{2}$ is Common

$$\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$$

$$\boxed{S_n = \frac{1}{2}n(n+1)}$$

For Natural numbers

↳ Positive whole numbers eg 1, 2, 3...

$\rightarrow \displaystyle\sum_{n=1}^{15} (2n+3)$ $\Rightarrow$ find the Sum of the first 15 terms of the series with n^{th} term $2n+3$

15 → last, n=15

n=1 → beginning, n=1

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$n = 15$

$a = 2(1)+3 = 5$

$d = (2(2)+3) - (2(1)+3) = 2$

2nd term − 1st term

$$\therefore S_{15} = \frac{15}{2}\left[2(5) + (15-1).2\right]$$

$$S_{15} = 285$$

$\rightarrow$ Sum of whole numbers between 1 to 100

$$S_n = \frac{1}{2}n(n+1)$$

$$S_{100} = \frac{1}{2}(100)(100+1)$$

$$\therefore S_{100} = 5{,}050$$

6.) Differatiation

$$\boxed{\frac{d}{dx}(x^n) = nx^{n-1}}$$

* multiply power
* minus 1 from power ~~and divide~~

Eg. $\frac{d}{dx}\frac{x^3}{5} = \frac{3x^2}{10}$

Eg. $y = 6x^2 + \frac{4}{\sqrt[3]{x}} - \frac{2}{x^2} + 1$

* Rationalise powers eg $\sqrt{2} = 2^{1/2}$

$$\frac{d}{dx}\, 6x^2 + 4x^{-1/3} - 2x^{-2}$$

$$\frac{dy}{dx} = 12x - \frac{4}{3\sqrt[3]{x^4}} + \frac{4}{x^3}$$

→ Gradient of a curve (or gradient of a tangent) at that point

Eg $y = x^2$ at $x = 1$ & $x = -2$

$$\frac{dy}{dx}x^2 = 2x$$

when $x = 1$ $\frac{dy}{dx} = 2(1)$

$\therefore$ gradient $= 2$ when $x = 1$

* differatiate ONCE and add the x value to get the gradient

when $x = -2$ $\frac{dy}{dx} = 2(-2)$

$\therefore$ gradient $= 4$ when $x = -2$

→ Tangents

* $\underline{\text{gradient of Curve = gradient of tangent}}$

Eg. $y = (4-x)(x+2)$ @ point $(2,8)$

find the tangent of the curve

→ * differentiate & find gradient

$$\frac{dy}{dx} \quad 8 + 2x - x^2 = 2 - 2x$$

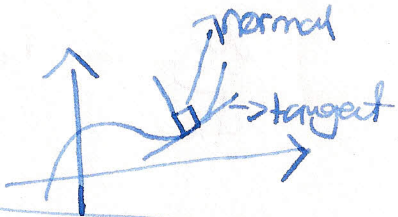
$$\text{at } x = 2, \ \frac{dy}{dx} = 2-(2) = \underline{\underline{-2}}$$

→ * Find the equation of the line (tangent)

$$y - y_1 = -2(x - x_1$$

$$y - 8 = -2(x-2) \Rightarrow y = -2x + 12$$

→ Normals = Right angles to a Curve.



$$\boxed{\text{Gradient of Normal} \times \text{Gradient of Curve} = -1}$$

Normal to $y = (4-x)(x+2)$ @ point $(2,8)$

* find gradient of Curve $\quad \frac{dy}{dx} = 2-(2) = -2$

$\therefore$ gradient of Normal $= \frac{-1}{-2} = \frac{1}{2}$

* Find equation of Normal

$$y - 8 = \frac{1}{2}(x-2)$$

$$2y - 16 = x - 2$$

$$\underline{\underline{2y = x + 14}}$$

7.) Integration

eg. $\int 2x\,dx = x^2 + c$ → constant

* add One to the power and divide

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + C}$$ → don't forget to add the Constant

→ Equation of the Curve

$$\frac{dy}{dx} = 6x(x-1) \qquad \text{point}(2,8)$$

* Integrate

$$\int 6x^2 - 6x\,dx$$

$$y = 2x^3 - 3x^2 + C$$ → don't forget Constant !!!

* find C , $\underset{(x,y)}{(2,8)}$

$$8 = 2(2)^3 - 3(2)^2 + c$$

$$\therefore C = 4$$

* Equation of the Curve

$$y = 2x^3 - 3x^2 + 4$$